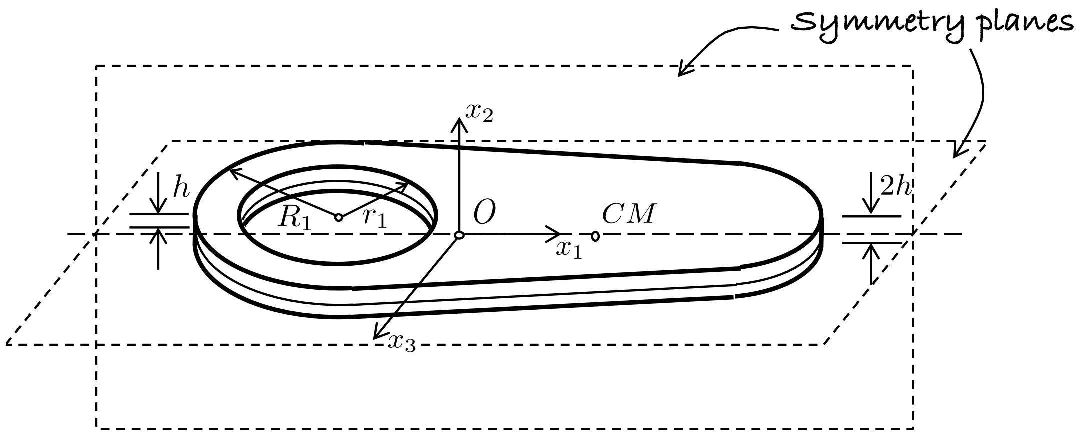
10. The three-dimensional, homogeneous object in the figure features two planes of symmetry x_1-x_2 and x_1-x_3. The origin O of the coordinate frame $\{x_1, x_2, x_3\}$ lies on the intersection of the two planes of symmetry, and it is offset from the center of mass CM. The x_1-axis is aligned with the intersection of the two planes of symmetry. What is the correct structure of the moment of inertia tensor $\mathbf{I}_O$ with respect to the $\{x_1, x_2, x_3\}$-coordinate frame? All entries are not zero unless indicated otherwise.



(a) $\mathbf{I}_0 = \begin{bmatrix} I_{11} & I_{12} & I_{13} \\ I_{12} & I_{22} & I_{23} \\ I_{13} & I_{23} & I_{33} \end{bmatrix}$

(b) $\mathbf{I}_0 = \begin{bmatrix} I_{11} & I_{12} & 0 \\ I_{12} & I_{22} & 0 \\ 0 & 0 & I_{33} \end{bmatrix}$

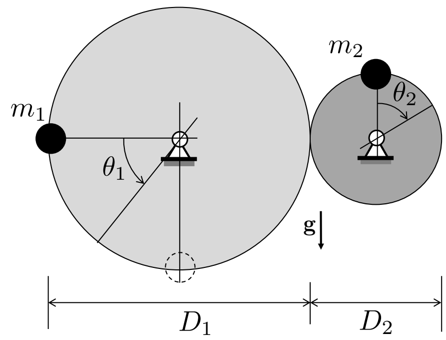
(c) $\mathbf{I}_0 = \begin{bmatrix} I_{11} & 0 & 0 \\ 0 & I_{22} & I_{23} \\ 0 & I_{23} & I_{33} \end{bmatrix}$

► (d) $\mathbf{I}_0 = \begin{bmatrix} I_{11} & 0 & 0 \\ 0 & I_{22} & 0 \\ 0 & 0 & I_{33} \end{bmatrix}$

(e) $\mathbf{I}_0 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & I_{33} \end{bmatrix}$

Solution: Since the reference frame is only translated along the principal axis x_1, the extra terms added by the Steiner's theorem appear only on the diagonal of the moment of inertia tensor.

17. Two particles of masses m_1 and m_2 are attached to two massless disks with diameters D_1 and D_2, where $D_1 = 2D_2$. The two disks roll without slipping at their contact point. The system is initially at rest in the shown configuration. What is the angular velocity $\dot{\theta}_1$ after a quarter rotation (i.e. when $\theta_1 = \frac{\pi}{2}$)?



(a) $\dot{\theta}_1 = \sqrt{\frac{4gD_1^2}{D_2^3}}$

(b) $\dot{\theta}_1 = \sqrt{\frac{gm_1}{D_2m_2}}$

(c) $\dot{\theta}_1 = \sqrt{\frac{g}{D_1}}$

► (d) $\dot{\theta}_1 = \sqrt{\frac{2g}{D_2}}$

(e) $\dot{\theta}_1 = \sqrt{\frac{8g(m_1+m_2)}{D_2(16m_1+m_2)}}$

Solution: Conservation of energy gives

$$\underbrace{T_{0,m_1}}_{=0} + V_{0,m_1} + \underbrace{T_{0,m_2}}_{=0} + V_{0,m_2} = T_{1,m_1} + \underbrace{V_{1,m_1}}_{=0} + T_{1,m_2} + \underbrace{V_{1,m_2}}_{=0}, \tag{32}$$

where T and V denote the kinetic and potential energy, respectively. Instants 0 and 1 denote the intial condition at rest and after a quarter rotation of m_1. If the origin of the coordinate system is chosen such that V_{1,m_1} and V_{1,m_2} are both zero we can write the energy balance as

$$m_1 g\frac{D_1}{2} + m_2 g D_2 = \frac{1}{2}\left(m_1\left(\frac{D_1}{2}\right)^2\dot{\theta}_1^2 + m_2\left(\frac{D_2}{2}\right)^2\dot{\theta}_2^2\right). \tag{33}$$

By making use of the fact that $D_1 = 2D_2$ and hence $\dot{\theta}_2 = \frac{D_1}{D_2}\dot{\theta}_1$ we

$$D_2 g(m_1 + m_2) = \frac{1}{2}D_2^2\dot{\theta}_1^2(m_1 + m_2), \tag{34}$$

which finally yields

$$\dot{\theta}_1 = \sqrt{\frac{2g}{D_2}}. \tag{35}$$