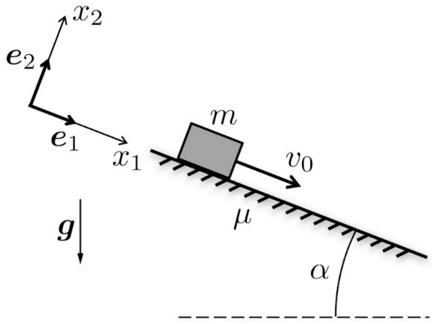
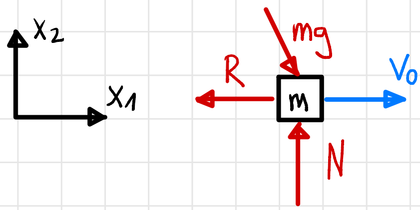
Problem 1: Sliding block I

A block of mass m slides along a rough inclined plane with kinetic friction coefficient μ, where α indicates the inclination angle. The block is given an initial velocity $\boldsymbol{v}_0$ at time $t_0 = 0$ directed down the incline. What is the time t_s after which the block comes to rest?

Given: v_0, α, μ, m, g



First let's draw a FBD:



Now use LMB in x_1 and x_2-direction:

$$x_1: \quad mg\sin(\alpha) - R = m a_1$$

$$x_2: \quad -mg\cos(\alpha) + N = 0 \quad \text{no movement in } x_2\text{-direction} \quad \Rightarrow \quad N = mg\cos(\alpha)$$

Since we have dynamic friction, we know $R = N \cdot \mu$

$$\Rightarrow \quad \cancel{m}g\sin(\alpha) - \mu \cdot \cancel{m}g\cos(\alpha) = \cancel{m}a_1$$

$$a_1 = g(\sin(\alpha) - \mu\cos(\alpha))$$

Now we can integrate a_1 to find $v_1(t)$:

$$v_1(t) = v_0 + \int_0^t a_1 \, dt = v_0 + gt(\sin(\alpha) - \mu\cos(\alpha))$$

And find t_{stop}, by requring $v_1(t_{stop}) = 0$

$$\Rightarrow \quad 0 = V_0 + g t_{stop} \left(\sin(\alpha) - \mu \cos(\alpha)\right)$$

$$\boxed{t_{stop} = \frac{V_0}{g\left(g\cos(\alpha) - \sin(\alpha)\right)}}$$

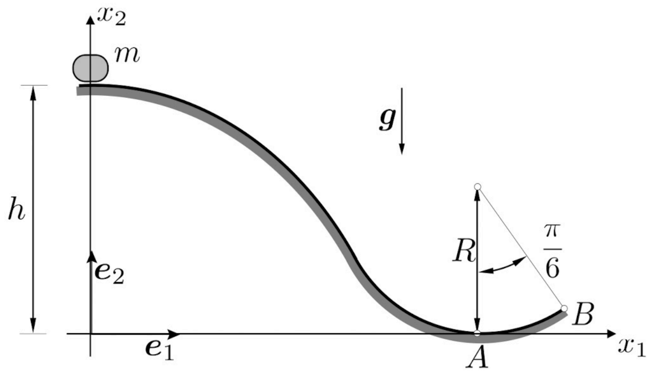
Notice: If $\mu \geq \tan(\alpha) \Rightarrow t_{stop} \to \infty$, so if we want the particle to stop we have to require $\mu < \tan(\alpha)$

(μ increases with α!)

Problem 2: Sliding block II

A block of mass m slides down a frictionless curved ground. The particle is released at an initial height h above the bottom of the loop (point A), with a negligible initial velocity. What is the speed v_B, with which the particle leaves the track at point B?

Given: m, h, R, g



To solve this problem we have to use the work-energy balance, as we have no knowledge of the exact path the particle takes.

$$T(t_2) - T(t_1) = W_{12} = \sum_i \int_{\underline{r}_1}^{\underline{r}_2} F_i \, d\underline{r}$$

If the system is conservative, we can simplify this a lot, so let's check:

$$F_g = -\frac{\partial mgx_1}{\partial x_1} = mg \quad \text{, so our } F_g \text{ is conservative.}$$

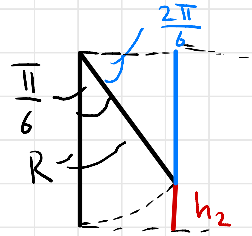
$$\Rightarrow \quad \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2 = mgh_1 - mgh_2$$

$$v_2 = ?, \quad v_1 = 0, \quad h_1 = h, \quad h_2 = ?$$

We can find h_2 from geometry:

$$\bullet = R\sin(2\pi/6) = R\,\frac{\sqrt{3}}{2}$$

$$h_2 = R - \bullet = R\left(1 - \frac{\sqrt{3}}{2}\right)$$



Now we can evaluate our WEB:

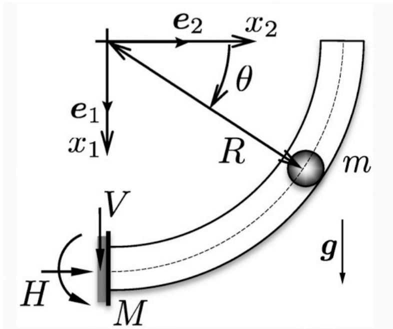
$$\frac{1}{2} m v_2^2 = mg\left(h - R\left(1 - \frac{\sqrt{3}}{2}\right)\right)$$

$$\underline{\underline{v_2 = \sqrt{2g\left(h - R\left(1 - \frac{\sqrt{3}}{2}\right)\right)}}}$$

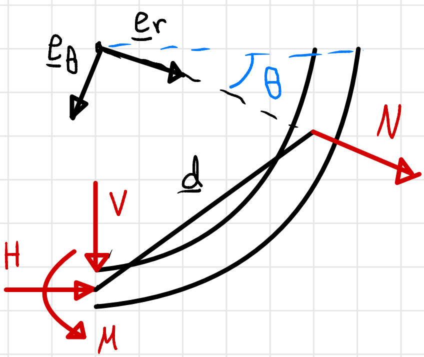
Problem 3: Guided sphere

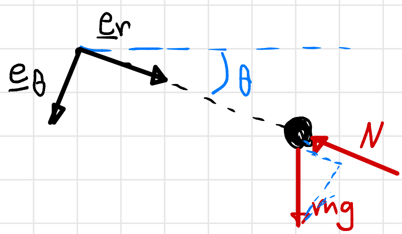
A particle of mass m and negligible dimensions is released from rest at $\theta = 0$ on a circular guide of radius R lying in a 2D plane. Gravity $\boldsymbol{g}$ acts downwards, as shown. The guide is fully clamped at $\theta = \pi/2$. What are the components H and V of the reaction force and the reaction moment M at the wall, as functions of the angular position θ?

Given: m, R, g



To solve this problem we first need two FBD's:





Firstly, find N using LMB:

$\underline{e}_r$: $\quad -N + mg\sin(\theta) = ma_r$

$$a_r = \frac{v^2}{R}$$

Find v^2 using WEB:

$$\frac{1}{2}mv^2 = mgR - (mgR - mgR\sin(\theta))$$

$\Rightarrow \quad v^2 = 2gR\sin(\theta)$

$\Rightarrow \quad N = 3mgR\sin(\theta)$

Now we can use LMB and AMB to find the reaction forces / torques

x_1: $\quad H = -N\cos(\theta) = -3mg\sin(\theta)\cos(\theta)$

x_2: $\quad V = -N\sin(\theta) = -3mg\sin^2(\theta)$

<u>AMB:</u> $\quad M\underline{e_3} = -(\underline{d} \times \underline{N})$

where $\quad \underline{d} = -R(1-\sin(\theta))\underline{e_1} + R\cos(\theta)\underline{e_2}$

and $\quad \underline{N} = N\sin(\theta)\underline{e_1} + N\cos(\theta)\underline{e_2}$

$\Rightarrow \quad M = 3mgR\sin(\theta)\cos(\theta)$